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Short-term reliability evaluation for power stations by using *Lz*-transform

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Abstract A short-term reliability evaluation is used for power stations, where each power generating unit is presented by a multi-state Markov model. The main obstacle for reliability evaluation in such a case is a "curse of dimensionality"—a great (huge) number of states of entire power station that should be analyzed. A modern approach is proposed based on using *Lz*-transform that drastically simplifies computation. The proposed approach is useful for power system security analysis and short-term operating decisions. In order to illustrate the proposed approach, the short-term reliability evaluation for a power station with different coal fired generating units is presented.

Keywords Power system, Generating unit, Generating capacity, Multi-State Markov model, *Lz*-transform, Universal generating function

1 Introduction

Many real world systems can perform their tasks with various distinguished levels of efficiency usually referred to as performance rates. A system that can have a finite number of performance rates is called the Multi-State System (MSS) [1, 2]. Multi-state models are widely used in the field of power system reliability assessment [3]. It has been recognized [4] that using simple two-state models for large generating units in generating capacity adequacy assessment can yield pessimistic appraisals. In order to more accurately assess power system reliability, many utilities now use multi-state models instead of two-state representations [4, 5]. A technique, called the apportioning method [4], is usually used to create steady-state multi-state generating unit models based on real world statistical data for generating units. Using this technique, steady-state probabilities of units residing at different generating capacity levels can be defined. When the short-term behavior of a MSS is studied, the investigation cannot be based on steady-state (long-term) probabilities. This investigation should use the MSS model in which transition intensities between any states of the model are known. One type of such model was suggested in [6]. In this study, transition intensities were defined for a simplified multi-state Markov model in which transitions to each derated state were possible from only one state with a nominal generating capacity. Such models of power generating units were also considered in [7]. In practice, transitions are possible among all states of the model. Such general model was presented in [8] where general multi-state Markov model was considered for coal fired generating unit. The paper described the method for the transition intensity estimation from actual unit failures (deratings) and repair statistics, which was presented by the observed realization of generating capacity stochastic process. The corresponding general multi-state Markov model was built based on this estimation.

It was shown that such important reliability indices as Loss of Load Probability (LOLP), Expected Energy Not Supplied (EENS) to consumers, etc, which were found for a short time, are essentially different from those found for a long-term reliability evaluation. Usually in each power station there are a number of generating units. If a power station consists of n generating units where each unit is represented by m-state Markov model, then the Markov model for the entire power station will have m^n states. In order to find reliability indices for entire power station, this

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model should be built and analyzed. As can be seen, it will require enormous efforts even for relatively small m and n. In order to avoid this obstacle in the paper an approach is suggested based on a new mathematical technique—*Lz*-transform [9]. A numerical example illustrates the application of the approach on power system reliability analysis and corresponding benefits.

2 Brief description of *Lz*-transform method

Lz-transform method is suggested for computation of short-term reliability indices of a power station consisting of numerous different generating units. *Lz*-transform was introduced in [9] where one can find its detailed description and corresponding mathematical proofs. Brief description of *Lz*-transform, the corresponding mathematical definitions and properties are presented in the Appendix A of this paper. The method for short-term reliability assessment of multi-state system is based on using *Lz*-transform.

Considering a discrete-state continuous-time (DSCT) Markov process, $G(t) \in \{g_1, g_2, \ldots, g_K\}$, which has K possible states where performance level associated with any state i $(1, 2, \ldots, K)$ is g_i. This Markov process is completely defined by a set of possible states $g = \{g_1, g_2, \ldots, g_K\}$, transition intensities matrix $A = |a_{ij}|$, $i, j = 1, 2, \ldots, K$ and by the probability distribution of initial states that can be presented by the corresponding set $p_0 = \{p_{10}, p_{20}, \ldots, p_{K0}\}$, where $p_1(t_0) = p_{10}, p_2(t_0) = p_{20}, \ldots, p_K(t_0) = p_{K0}$.

We designate $p_i(t)$ $(i = 1, 2, \ldots, K)$, as the probability of process $G(t)$ staying in state i at instant $t \geq 0$.

By solving the following system of ordinary linear differential equations [10]

$$\begin{cases} \frac{dp_1(t)}{dt} = a_{11}(t)p_1(t) + a_{12}(t)p_2(t) + \cdots + a_{1K}(t)p_K(t) \\ \frac{dp_2(t)}{dt} = a_{21}(t)p_1(t) + a_{22}(t)p_2(t) + \cdots + a_{2K}(t)p_K(t) \\ \vdots \\ \frac{dp_K(t)}{dt} = a_{K1}(t)p_1(t) + a_{K2}(t)p_2(t) + \cdots + a_{KK}(t)p_K(t) \end{cases} \tag{1}$$

under given initial conditions $p_0 = \{p_{10}, p_{20}, \ldots, p_{K0}\}$, one can find all the probabilities $p_i(t)$, $i = 1, 2, \ldots, K$.

In according to [9] (see also the Appendix) *Lz*-transform of a DSCT Markov process $G(t)$ is a function defined as follows:

$$Lz\{G(t)\} = \sum_{i=k}^{K} p_i(t) z^{g_i} \tag{2}$$

where z is a complex variable in general case.

Any element j in MSS can have k_j different states corresponding to different performances, represented by the set $i = 1, 2, \ldots, k_j$, where g_{ji} is the performance rate of element j in the state i, $i = 1, 2, \ldots, k_j$, and $j \in \{1, 2, \ldots, n\}$, where n is the number of elements in the MSS.

At first stage a Markov model of stochastic process should be built for each multi-state element in MSS. Based on the model, state probabilities

$$p_{ji}(t) = \Pr\{G_j(t) = g_{ji}\}, \quad i = 1, 2, \ldots, k_j \tag{3}$$

for every MSS element can be obtained as a solution of the corresponding system of differential (1) under given initial conditions.

Then individual *Lz*-transform for each element j should be found

$$Lz\{G_j(t)\} = \sum_{i=1}^{k_j} p_{ji}(t) z^{g_{ji}}, \quad j = 1, 2, \ldots, n \tag{4}$$

All MSS elements are arranged in a MSS structure by using system structure function f, which produces the output stochastic performance process of the entire MSS based on the stochastic processes of all MSS elements:

$$G(t) = f(G_1(t), G_2(t), \ldots, G_n(t)) \tag{5}$$

Lz-transform of the output stochastic process $G(t)$ for the entire MSS should be defined based on the previously determined *Lz*-transform for each element j and system structure function f.

In [9] it was shown that in order to find *Lz*-transform of the resulting DSCT Markov process $G(t)$, which was the single-valued function $G_t = f(G_1(t), G_2(t), \ldots, G_n(t))$ of n independent DSCT Markov processes $G_j(t), j = 1, 2, \ldots, n$, one can apply Ushakov's Universal Generating Operator (UGO) to all individual *Lz*-transforms $Lz\{G_j(t)\}$ over all time points $t \geq 0$.

$$Lz\{G(t)\} = \Omega_f \quad \{Lz\{G_1(t)\}, Lz\{G_2(t)\}, \ldots, Lz\{G_n(t)\}\} \tag{6}$$

So, by using Ushakov's operator Ω_f over all *Lz*-transforms of individual elements, one can obtain the resulting *Lz*-transform $Lz\{G(t)\}$ associated with output performance stochastic process $G(t)$ of the entire MSS.

The technique of Ushakov's operator applying is well established for many different structure functions f [2]. By using the technique, the computational burden (6) decreases drastically.

The resulting *Lz*-transform $Lz\{G(t)\}$ is associated with the output performance stochastic process for the entire MSS. MSS reliability measures can be easily derived from the resulting *Lz*-transform $Lz\{G(t)\}$.

3 Reliability analyses for power stations

In Fig. 1 one can see a power station consisting of n generating units connected to a common switchgear.

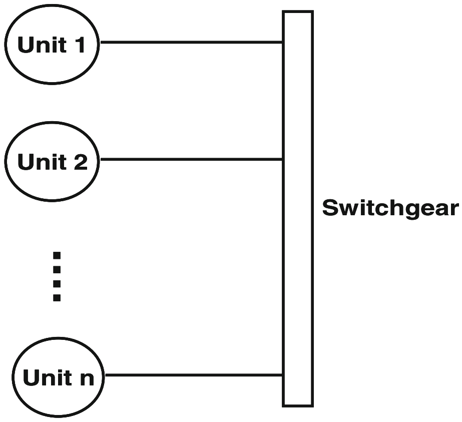

Fig. 1 Power system consisting of n generating units connected to switchgear

Each generating unit i, $i = 1, 2, \ldots, n$ is described by discrete-state continuous-time Markov process $G_i(t)$. The number of states is m_i and g_{ij} is a generating capacity of unit i in state j, $j = 1, 2, \ldots, m_i$. We designate transition rates (intensities) for transition from state c to state d for unit i as $a_{cd}^{(i)}$.

The switchgear is described by a two-state Markov model with capacities $g_{s1} = 0$ (complete failure) and $g_{s2} \geq \sum_{i=1}^{n} g_{im_i}$, and transition rates $a_{12}^{(s)}$, $a_{21}^{(s)}$.

For generating unit number i there is the following system of differential equations in order to find probabilities $p_{ij}(t)$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m_i$ that the unit i will be in state j at instant $t \geq 0$.

$$\begin{aligned}
\frac{\mathrm{d}p_{i1}(t)}{\mathrm{d}t} &= a_{11}^{(i)}(t)p_{i1}(t) + a_{12}^{(i)}(t)p_{i2}(t) + \cdots + a_{im_i}^{(i)}(t)p_{im_i}(t) \\
\frac{\mathrm{d}p_{i2}(t)}{\mathrm{d}t} &= a_{21}^{(i)}(t)p_{i1}(t) + a_{22}^{(i)}(t)p_{i2}(t) + \cdots + a_{2m_i}^{(i)}(t)p_{im_i}(t) \\
&\vdots \\
\frac{\mathrm{d}p_{im_i}(t)}{\mathrm{d}t} &= a_{m_i1}^{(i)}(t)p_{i1}(t) + a_{m_i2}^{(i)}(t)p_{i2}(t) + \cdots + a_{m_im_i}^{(i)}(t)p_{im_i}(t)
\end{aligned} \tag{7}$$

The system (7) is solved under specified initial conditions p_{i0}.

By using expression (4) individual Lz-transforms is found for each Markov process $G_i(t)$, associated with generating capacity of unit i

$$Lz\{G_i\{t\}\} = \sum_{j=1}^{m_i} p_{ij}(t)z^{g_{ij}}, \quad i = 1, 2, \ldots, n \tag{8}$$

Analogously, Lz-transform for Markov process $G_S(t)$ associated with switchgear can be obtained

$$Lz\{G_S(t)\} = p_{s2}z^{g_{s2}} + p_{s1}z^{g_{s1}} \tag{9}$$

At the next step based on these individual Lz-transforms, Lz-transform $Lz\{G_Y(t)\}$ of resulting Markov process $G_Y(t)$ should be calculated by applying Ushakov's Universal Generating Operator (UGO) to all individual Lz-transforms $Lz\{G_j(t)\}$ over all time points $t \geq 0$,

$$\begin{aligned} Lz\{G_Y(t)\} = \Omega_f\{&Lz\{G_1(t)\}, Lz\{G_2(t)\}, \ldots, \\ &Lz\{G_n(t)\}, Lz\{G_S(t)\}\} \end{aligned} \tag{10}$$

where f is the power system structure function.

For a power system, it is depicted in Fig. 1,

$$f = \min\{G_1(t) + G_2(t) + \cdots + G_n(t), G_S(t)\} \tag{11}$$

because all units are connected in parallel and the switchgear connected in series with all these units.

Therefore, in according to [2] we obtain

$$\begin{aligned} Lz\{G_Y(t)\} = \Omega_{f\,\mathrm{ser}}\{&\Omega_{\mathrm{par}}\{Lz\{G_1(t)\}, Lz\{G_2(t)\}, \ldots, \\ &Lz\{G_n(t)\}\}, Lz\{G_S(t)\}\} \end{aligned} \tag{12}$$

where Ω_{par}, Ω_{ser} are Ushakov's generating operators for elements connected in parallel and in series respectively,

$$\begin{aligned} &\Omega_{\mathrm{par}}\{Lz\{G_1(t)\}, Lz\{G_2(t)\}, \ldots, Lz\{G_n(t)\}\} \\ &\quad = Lz\{G_1(t)\} \times Lz\{G_2(t)\} \times \cdots \times Lz\{G_n(t)\} \\ &\quad = \prod_{i=1}^{n}\sum_{j=1}^{m_i} p_{ij}(t)z^{g_{ij}} = \sum_{i=1}^{r} q_i(t)z^{x_i} \end{aligned} \tag{13}$$

$$\begin{aligned} &\Omega_{f\,\mathrm{ser}}\left\{\sum_{i=1}^{r} q_i(t)z^{x_i}, \sum_{j=1}^{2} p_{sj}(t)z^{g_{sj}}\right\} \\ &\quad = \sum_{i=1}^{r}\sum_{j=1}^{2} q_i(t)p_{sj}(t)z^{min\{x_i, g_{sj}\}} \end{aligned} \tag{14}$$

Therefore, Lz-transform for output stochastic process $G_Y(t)$ will be the following

$$Lz\{G_Y(t)\} = \sum_{i=1}^{r}\sum_{j=1}^{2} q_i(t)p_{sj}(t)z^{min\{x_i, g_{sj}\}} = \sum_{k=1}^{K} p_k(t)z^{y_k} \tag{15}$$

If Lz-transform $Lz\{G_Y(t)\} = \sum_{k=1}^{K} p_k(t)z^{y_k}$ of the entire MSS output stochastic process $G_Y(t) \in \{y_1, y_2, \ldots, y_K\}$ is known, then the important system reliability measures can be easily found.

The power station availability for demand level w is treated as system ability to provide power supply to consumers with summarized load w. It means that power station should be in states with generating capacity more or equal w.

So, the system availability for the constant demand level w at instant $t \geq 0$

$$p_{Aw}(t) = \sum_{y_k \geq w} p_k(t) \tag{16}$$

To find MSS instantaneous availability one should summarize all probabilities in Lz-transform from terms where powers of z are greater or equal to demand w.

The value $1 - p_{Aw}(t)$ characterizes a loss of load probability ($LOLP_w$) for a given demand level w

$$LOLP_w(t) = 1 - p_{Aw}(t) \tag{17}$$

The expected generating capacity deficiency (ECD_w) of the system is expressed by the following function:

$$ECD_w(t) = \sum_{k=1}^{K} p_k(t)(w - y_k)1(w - y_k) \tag{18}$$

where

$$\begin{cases} 1(w - y_k) = 1, \text{ if } (w - y_k) > 0 \\ 1(w - y_k) = 0, \text{ if } (w - y_k) \le 0 \end{cases} \tag{19}$$

Based on the calculated function $ECD_w(t)$, the expected energy not supplied to consumers during time $t(EENS_w(t))$ can be computed:

$$EENS_w(t) = \int_0^t ECD(u)\mathrm{d}u \tag{20}$$

All reliability measures depend strongly on the initial conditions, under which the system of differential equations for each generating unit should be solved. In other words, reliability measures for a power system depend strongly on initial states of units.

4 Numerical example

As an example we consider a power station consisting of 3 coal fired generating units U1, U2 and U3. U1 has nominal generating capacity of 580 MW, and U2 and U3 are the same and each has nominal generating capacity 400 MW.

U1 is presented by 4-state Markov model with corresponding capacity levels $g_{11} = 0$, $g_{12} = 300$ MW, $g_{13} = 480$ MW, and $g_{14} = 580$ MW, the transition intensity matrix is decribed as

$$A_1 = |a_{ij}^{(1)}| = \begin{vmatrix} -0.0289 & 0.0289 & 0 & 0 \\ 0.1628 & -0.4652 & 0.2791 & 0.0233 \\ 0 & 0.0235 & -0.2019 & 0.1784 \\ 0.0001 & 0.0001 & 0.0008 & -0.0010 \end{vmatrix}$$

Each element in matrix A_1 is represented by such units as 1/h.

Units U2 and U3 are the same and presented by 4-state Markov models with corresponding capacity levels $g_{21} = g_{31} = 0$, $g_{22} = g_{32} = 200$ MW, $g_{23} = g_{33} = 310$ MW, and $g_{24} = g_{34} = 400$ MW and corresponding transition intensity matrices

$$A_2 = A_3 = \left|a_{ij}^{(2)}\right| = \begin{vmatrix} -0.1059 & 0.1059 & 0 & 0 \\ 0.0291 & -0.1456 & 0.0971 & 0.0194 \\ 0 & 0.0173 & -0.2890 & 0.2717 \\ 0.0002 & 0.0001 & 0.0010 & -0.0013 \end{vmatrix}$$

Each element in matrix A_2 is represented by such units as 1/h.

The switchgear is presented by binary-state model with states $g_{s1} = 0$ and $g_{s2} = 1{,}400$ MW, the transition intensity matrix is described as

$$A_S = \left|a_{ij}^{(s)}\right| = \begin{vmatrix} -0.042 & 0.042 \\ 0.00011 & -0.00011 \end{vmatrix}$$

Each element in matrix A_S is represented by such units as 1/h.

4.1 Questions

In according to long-term maintenance plan at time instant t_0, generating unit U3 should be shut down in order to perform a preventive maintenance.

The question is described as follows: can U3 be shut down, if at time instant t_0 unit U1 is in state 4 with nominal capacity $g_{14} = 580$ MW, unit U2 is in state 2 with generating capacity $g_{22} = 200$ MW and the switchgear at time instant t_0 is in state 2?

The required demand level $w = 690$ MW and probability (risk) of loss of load should be less than 0.05 or, in other words, the power system availability should be greater than the required value $p_{A690}^{(r)} = 0.95$.

4.2 Solutions

Remark 1 When unit U3 will be shut down, the remaining generating capacity of unit U1 and unit U2 is as follows: $g_{14} + g_{22} = 780$ MW $> w = 690$ MW. It means that formally generating capacity of the power station is enough to provide the required demand. The question is that the corresponding risk of loss of load can be estimated only by dynamic (short-time) reliability evaluation. Time instant t_0 will be determined as initial point $t_0 = 0$ for the calculation.

In according to Lz-transform method at first step state probabilities $p_{ij}(t)$, $i = 1, 2$; $j = 1, 2, 3, 4$ of state j at any time instant $t \ge 0$ for generating units U1 ($i = 1$) and U2 ($i = 2$) can be found by solving the following system of differential equations

$$\begin{aligned}
\frac{\mathrm{d}p_{i1}(t)}{\mathrm{d}t} &= -(a_{12}^{(i)} + a_{13}^{(i)} + a_{14}^{(i)})p_{i1}(t) + a_{21}^{(i)}p_{i2}(t) \\
&\quad + a_{31}^{(i)}p_{i3}(t) + a_{41}^{(i)}p_{i4}(t) \\
\frac{\mathrm{d}p_{i2}(t)}{\mathrm{d}t} &= a_{12}^{(i)}p_{i1}(t) - (a_{21}^{(i)} + a_{23}^{(i)} + a_{24}^{(i)})p_{i2}(t) \\
&\quad + a_{32}^{(i)}p_{i3}(t) + a_{42}^{(i)}p_{i4}(t) \\
\frac{\mathrm{d}p_{i3}(t)}{\mathrm{d}t} &= a_{13}^{(i)}p_{i1}(t) + a_{23}^{(i)}p_{i2}(t) - (a_{31}^{(i)} + a_{32}^{(i)} \\
&\quad + a_{34}^{(i)})p_{i3}(t) + a_{43}^{(i)}p_{i4}(t) \\
\frac{\mathrm{d}p_{i4}(t)}{\mathrm{d}t} &= a_{14}^{(i)}p_{i1}(t) + a_{12}^{(i)}p_{i2}(t) + a_{13}^{(i)}p_{i3}(t) \\
&\quad - (a_{41}^{(i)} + a_{42}^{(i)} + a_{43}^{(i)})p_{i4}(t)
\end{aligned}$$

for $i = 1$ (unit U1) and for $i = 2$ (unit U2).

For $i = 1$ the system should be solved under initial conditions $p_{11}(0) = p_{12}(0) = p_{13}(0) = 0,\ p_{14}(0) = 1$ and for $i = 2$ the system should be solved under initial conditions $p_{21}(0) = p_{23}(0) = p_{24}(0) = 0, p_{22}(0) = 1$, because at time instant t_0 unit U1 is in state 4 and unit U2 is in state 2. From now on, we shall write initial conditions as a corresponding vector of initial probabilities. For example, if unit U1 at time instant $t = 0$ will be in state 4, we shall write the following U1 [0001]. If unit U2 at instant $t = 0$ will be in state 2, we shall write the following U2 [0100].

Based on the probabilities $p_{ij}(t)$, which are found by solving of the corresponding system of differential equations under the given initial condition, Lz-transforms for generating capacity processes $G_1(t)$ and $G_2(t)$ associated with unit U1 and unit U2 can be obtained, respectively,

$$Lz\{G_1(t)\} = p_{11}(t)z^0 + p_{12}(t)z^{300} + p_{13}(t)z^{480} + p_{14}(t)z^{580}$$

$$\begin{aligned}
Lz\{G_2(t)\} &= p_{21}(t)z^0 + p_{22}(t)z^{200} + p_{23}(t)z^{310} \\
&\quad + p_{24}(t)z^{400}
\end{aligned}$$

State probabilities $p_{sj}(t), j = 1, 2$ of state j at any time instant $t \geq 0$ for the switchgear can be found by solving the following system of differential equations

$$\begin{aligned}
\frac{\mathrm{d}p_{s1}(t)}{\mathrm{d}t} &= -a_{12}^{(s)}p_{s1}(t) + a_{21}^{(s)}p_{s2}(t) \\
\frac{\mathrm{d}p_{s2}(t)}{\mathrm{d}t} &= a_{12}^{(s)}p_{s1}(t) - a_{21}^{(s)}p_{s2}(t)
\end{aligned}$$

The switchgear at time instant t_0 is in state 2 and, therefore, the last system of differential equations should be solved under the following initial conditions: $p_{s1}(0) = 0, p_{s2}(0) = 1$. After solving of the system Lz-transform for capacity stochastic process $G_S(t)$ associated with switchgear can be written as

$$Lz\{G_S(t)\} = p_{s1}(t)z^0 + p_{s2}(t)z^{1,400}$$

The system structure function is follows:

$$f = \min\{G_1(t) + G_2(t), G_S(t)\}$$

In according to (12) we obtain Lz-transform for output stochastic capacity process $G_Y(t)$ as follows:

$$Lz\{G_Y(t)\} = \Omega_{f\mathrm{ser}}\{\Omega_{\mathrm{par}}\{Lz\{G_1(t)\}, Lz\{G_2(t)\}\}, Lz\{G_S(t)\}\}$$

Performing computation accordingly with expressions (13), (14) and (15), we obtain Lz-transform of output process $G_Y(t)$

$$\begin{aligned}
Lz\{G_Y(t)\} &= p_{s2}(t)p_{14}(t)p_{24}(t)z^{980} + p_{s2}(t)p_{14}(t)p_{23}(t)z^{890} \\
&\quad + p_{s2}(t)p_{13}(t)p_{24}(t)z^{880} + p_{s2}(t)p_{13}(t)p_{23}(t)z^{790} \\
&\quad + p_{s2}(t)p_{14}(t)p_{22}(t)z^{780} + p_{s2}(t)p_{12}(t)p_{24}(t)z^{700} \\
&\quad + p_{s2}(t)p_{13}(t)p_{22}(t)z^{680} + p_{s2}(t)p_{12}(t)p_{23}(t)z^{610} \\
&\quad + p_{s2}(t)p_{14}(t)p_{21}(t)z^{580} + p_{s2}(t)p_{12}(t)p_{22}(t)z^{500} \\
&\quad + p_{s2}(t)p_{13}(t)p_{21}(t)z^{480} + p_{s2}(t)p_{11}(t)p_{24}(t)z^{400} \\
&\quad + p_{s2}(t)p_{11}(t)p_{23}(t)z^{310} + p_{s2}(t)p_{12}(t)p_{21}(t)z^{300} \\
&\quad + p_{s2}(t)p_{11}(t)p_{22}(t)z^{200} + (p_{s2}(t)p_{11}(t)p_{21}(t) \\
&\quad + p_{s1}(t))z^0
\end{aligned}$$

In according to (16), we obtain the power station availability for demand level $w = 690$ MW

$$\begin{aligned}
p_{Aw}(t) &= p_{s2}(t)p_{14}(t)p_{24}(t) + p_{s2}(t)p_{14}(t)p_{23}(t) + \\
&\quad p_{s2}(t)p_{13}(t)p_{24}(t) + p_{s2}(t)p_{13}(t)p_{23}(t) + \\
&\quad p_{s2}(t)p_{14}(t)p_{22}(t) + p_{s2}(t)p_{12}(t)p_{24}(t)
\end{aligned}$$

Graphs of function $p_{Aw}(t)$ for 4 different initial conditions of units U1 and U2 are shown in Fig. 2.

Remind that designation U1 [0001], U2 [0001] means that unit U1 and unit U2 at time instant $t = 0$ are both in the state 4; designation U1 [0100], U2 [0001] means that unit U1 at time instant $t = 0$ is in state 2 and unit U2 is in state 4; etc.

A curve that we are interested in is the curve for the given initial conditions U1 [0001], U2 [0100]. It can be seen from Fig. 2, the required power system availability is

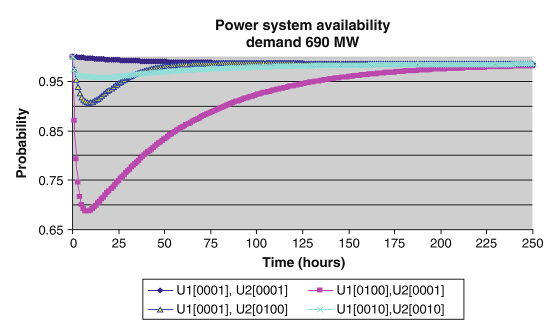

Fig. 2 Power system availability as function of time for different initial conditions of the units

not provided in this case. The minimum of availability function is almost 0.90.

So, unit U3 cannot be shut down in order to perform preventive maintenance actions, if at this time instant unit U1 is in state 4 and unit U2 is in state 2.

It can be seen from Fig. 2, long term power system availability

$$p_{A690} = \lim_{t\to\infty} p_{A690}(t) = 0.985$$

is essentially different from the calculated short-term availability, which is also strongly depends on initial conditions of the units. The transient mode is almost finished within 250 h. All the curves in Fig. 1 aspire eventually to the value $p_{A690} = 0.985$.

In addition, by using expressions (18) and (19) one can easily compute expected capacity deficiency for all possible initial conditions of units. The computed results can be seen in Fig. 3.

The maximum of function $ECD_w(t)$ determines the load that expected to be cut by load shedding system to prevent generating unit overload.

As in the previous case, long term ECD_{690} of power station

$$ECD_{690} = \lim_{t\to\infty} ECD_{690}(t) = 5 \text{ MW}$$

is essentially different from the calculated short-term expected capacity deficiency $ECD_{690}(t)$. For example, maximum $ECD_{690}(t)$ that was calculated for the initial conditions U1 [0100], U2 [0001] is equal 91 MW and it is much greater than long term value 5 MW.

Remark 2 In order to solve the problem by using straightforward Markov method a model with $4 \times 4 \times 2 = 32$ states is built to solve a system with 32 differential equations.

By using *Lz*-transform method we have to solve two systems of 4 differential equations and one system of 2 differential equations. All other calculations use only simple algebra.

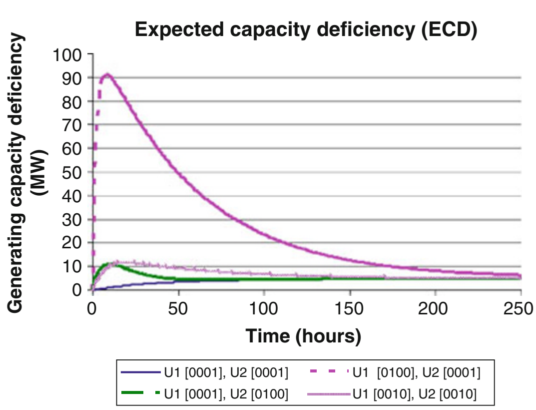

Fig. 3 Expected capacity deficiency as function of time and initial conditions of units

5 Conclusion

A *Lz*-transform method is suggested for a short-term reliability analysis of power stations. Evaluation of such important reliability indices as power system availability, expected capacity deficiency, the expected energy not supplied to consumers, are considered. The method application decreases drastically a computation burden compared with the straightforward Markov method. It is shown that short-term reliability indices are essentially different from long-term (steady-state) indices. So, operative decisions for power system cannot be based on long-term indices. They should be based on short-term reliability evaluation.

In order to illustrate the suggested approach, a numerical example is presented. The method is applied for a short-term reliability analysis of a power station consisting of coal fired power generating units and switchgear.

Appendix A

Lz-transform: definition and main properties

Here we present the definition and main properties of *Lz*-transform. The description will follow the work [9].

We consider a discrete-state continuous-time (DSCT) Markov process [10] (Trivedi 2002) $X(t) \in \{x_1, x_2, \ldots, x_K\}$, $t \geq 0$, which has K possible states i ($i = 1, 2, \ldots, K$), where performance level associated with any state i is x_i. This Markov process is completely defined by set of possible states $x = \{x_1, x_2, \ldots, x_K\}$, transition intensity matrix $A = |a_{ij}(t)|$, $i, j = 1, 2, \ldots, K$ and by initial state probability distribution that can be presented by corresponding set

$$p_0 = \{p_{10} = \Pr\{X(0) = x_1\}, p_{20} = \Pr\{X(0) = x_2\}, \ldots,$$
$$p_{K0} = \Pr\{X(0) = x_K\}\}$$

From now on, we shall use for such Markov process following notation by triplet:

$$X(t) = \{x, A, p_0\}$$

Remark A.1 If functions $a_{ij}(t) = a_{ij}$ are constants, then the DSCT Markov process is said to be *time-homogeneous*. When $a_{ij}(t)$ are time dependent, the resulting Markov process is *non-homogeneous*.

Definition A.1 *Lz-transform of a discrete-state continuous-time Markov process $X(t) = \{x, A, p_0\}$ is a function $u(z,t,p_0)$ defined as*

$$Lz\{X(t)\} = u(z,t,p_0) = \sum_{i=1}^{K} p_i(t) z^{x_i}$$

where $p_i(t)$ is a probability that the process is in state i at time instant $t \geq 0$ for any given initial state probability distribution p_0, and z is a complex variable in general case.

In the future we sometime shall omit symbol p_0 and write simply $u(z,t)$ keeping in mind that Lz-transform will depend on initial probability distribution p_0.

Example Consider a simplest element which has only two states 1 and 2 with corresponding performance levels $x_1 = 0$ and $x_2 = x_{\text{nom}}$, respectively. It means that state 1 is a complete failure state and state 2 is a state with nominal performance. The element failure rate is λ and the repair rate is μ.

Suppose that at time instant $t = 0$ the element is in the state 2, so that initial state probability distribution is follows: $p_0 = \{p_{10}, p_{20}\} = \{p_1(0), p_2(0)\} = \{0, 1\}$.

Let's define Lz-transform for Markov process $X(t)$ that describes the element's behavior.

Solution The Markov process $X(t)$ for our example is defined by the triplet $X(t) = \{x, A, p_0\}$, where x, A, p_0 are defined:

1) set of possible states $x = \{x_1, x_2\} = \{0, x_{\text{nom}}\}$.
2) transitions intensities matrix $A = |a_{ij}| = \begin{vmatrix} -\mu & \mu \\ \lambda & -\lambda \end{vmatrix}$, $i, j = 1, 2$.
3) initial states probability distribution $p_0 = \{p_{10}, p_{20}\} = \{0, 1\}$.

Therefore, states probabilities of the process $X(t)$ at any time instant $t \geq 0$ will be defined as a solution of the following system of differential equations

$$\begin{cases} \frac{\mathrm{d}p_1(t)}{\mathrm{d}t} = -\mu p_1(t) + \lambda p_2(t) \\ \frac{\mathrm{d}p_2(t)}{\mathrm{d}t} = \mu p_1(t) - \lambda p_2(t) \end{cases}$$

under initial conditions $p_1(0) = p_{10} = 0; p_2(0) = p_{20} = 1$.

After solving the system one obtains

$$p_2(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)t}$$

$$p_1(t) = \frac{\mu}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)t}$$

So, in according to Definition A.1 one can obtain Lz-transform of the given Markov process as follows:

$$Lz\{X(t)\} = u(z,t,p_0) = \sum_{i=1}^{2} p_i(t) z^{x_i}$$

$$= \left[\frac{\lambda}{\mu+\lambda} - \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)t}\right] z^0 + \left[\frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)t}\right] z^{x_{\text{nom}}}$$

A1 Existence and uniqueness of Lz-transform

Each discrete-state continuous-time Markov process under certain initial conditions has only one (unique) Lz-transform $u(z,t)$ and each Lz-transform $u(z,t)$ will have only one corresponding DSCT Markov process $X(t)$ developing from these initial conditions.

We'll formulate this as an *existence and uniqueness property* of Lz-transform.

Proposition A1.1 Each discrete-state continuous-time Markov process $X(t)$ under certain initial conditions p_0 has one and only one Lz-transform: $Lz\{X(t)\} = u(z,t,p_0)$.

Remark A1.2 The inverse statement is also true: if it is known $u(z,t,p_0) = \sum_{i=1}^{K} p_i(t) z^{x_i}$, where $p_i(t)$ are defined as a solution of the system (8) (where coefficients $a_{kl}(t), k, l = 1, 2, \ldots, K$ are continuous functions of time t) under initial conditions (9), then exists only unique DSCT Markov process $X(t)$, for which

$$Lz\{X(t)\} = u(z,t,p_0) = \sum_{i=1}^{K} p_i(t) z^{x_i}$$

Remark A1.3 In reliability interpretation Lz-transform may be applied to an aging system and to a system at burn-in period as well as to a system with constant failure and repair rates. The unique condition that should be fulfilled is a continuity of transition intensities $a_{ij}(t)$.

A2 Main properties of Lz-transform

Property A2.1 Multiplying DSCT Markov process on constant value a is equal to multiplying corresponding performance level x_i at each state i on this value

$$Lz\{aX(t)\} = \sum_{i=1}^{K} p_i(t) z^{ax_i}$$

Property A2.2 *Lz*-transform from a single-valued function $f(G(t), W(t))$ of two independent DSCT Markov processes $G(t)$ and $W(t)$ can be found by applying Ushakov's universal generating operator Ω_f to *Lz*-transform from $G(t)$ and $W(t)$ processes over all time points $t \geq 0$

$$Lz\{f(G(t), W(t))\} = \Omega_f\{Lz\{G(t)\}, Lz\{W(t)\}\}$$

The property provides *Lz*-transform application to multi-state system reliability analysis.

References

[1] Natvig B (2011) Multi-state systems reliability theory with applications. John Wiley & Sons, New York, USA
[2] Lisnianski A, Frenkel I, Ding Y (2010) Multi-state system reliability analysis and optimization for engineers and industrial managers. Springer, Heidelberg, Germany
[3] Billinton R, Allan R (1996) Reliability evaluation of power system. Plenum Press, New York, USA
[4] Billinton R, Li W (2004) Incorporating multistate unit models in composite system adequacy assessment. In: Proceedings of the International Conference on Probabilistic Methods Applied to Power Systems (PMAPS'04), Ames, IA, USA, 12–16 Sept 2004, pp 70–75
[5] Reshid M, Abd Majid M (2011) A multi-state reliability model for a gas fueled cogenerated power plant. J Appl Sci 11(11): 1945–1951
[6] Goldner SH (2006) Markov model for a typical 360 MW coal fired generation unit. Commun Depend Qual Manag 9(1):24–29
[7] Ding Y, Lisnianski A, Wang P et al (2009) Dynamic reliability assessment for bilateral contract electricity providers in restricted power systems. Electr Power Syst Res 79:1424–1430
[8] Lisnianski A, Elmakias D, Laredo D et al (2012) A multi-state Markov model for a short-term reliability analysis of a power generating unit. Reliab Eng Syst Safe 98(1):1–6
[9] Lisnianski A (2012) *Lz*-transform for a discrete-state continuous-time Markov process and its applications to multi-state reliability. In: Lisnianski A, Frenkel I (eds) Recent Advances in System Reliability: Signatures. Multi-state Systems and Statistical Inference, Springer, London, UK, pp 79–96
[10] Trivedy K (2002) Probability and statistics with reliability queuing and computer science applications. Wiley, New York, USA

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